Improving performance of droop-controlled microgrids through distributed PI-control

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Background: Microgrids facilitate transition to distributed power generation

- Networks with distributed generation (DG) units, loads and energy storage
- Local, autonomous operation in grid-connected or islanded mode
- Synchrony and power balance rely on control of power electronic inverters at DG units

Setup: Droop-controlled inverter network

- Network \( G = \{ V, E \} \), with \(|V| = N\) inverters, connected by lines \( E \) = \( \{ e_i \} \)
- Frequency droop control:
  \[
  \tau \dot{\omega}_i = -\omega_i + \omega - k_i(P_i - P_i^0),
  \]
  \( k_i \): droop gain, \( P_i \): active power injection, \( \tau \): time constant, \( \omega \): synchronous frequency, \( P_i^0 \): setpoints for frequency and active power.
- Power flow (linearized):
  \[
  P_i \approx \sum_{j \in N_i} b_{ij}(\theta_i - \theta_j),
  \]
- Closed loop system dynamics, driven by disturbance \( w \)
  \[
  \frac{d}{dt} \begin{bmatrix} \theta_i \\ \omega_i \end{bmatrix} = \begin{bmatrix} 0 & I \\ -L_d & -L_i \end{bmatrix} \begin{bmatrix} \theta_i \\ \omega_i \end{bmatrix} + \begin{bmatrix} 0 \\ H \end{bmatrix} w
  \]
- \( L_d, L_i \): reactance matrices (weighted graph Laplacian).

Issue: Droop control causes steady-state frequency \( \omega^s \) to deviate from nominal value \( \omega \).

\[ \Rightarrow \text{motivates secondary control: } \]

Centralized averaging PI control (CAPI):

- \( \tau \dot{\omega}_i = [\text{droop control}] + \Omega \)
- \( q \Omega_i = -\omega_i - \sum_{j \in N_i} c_{ij}(\Omega_i - \Omega_j) \)

Distributed averaging PI control (DAPI):

- \( \tau \dot{\omega}_i = [\text{droop control}] + \Omega_i \)
- \( q \Omega_i = -\omega_i - \sum_{j \in N_i} c_{ij}(\Omega_i - \Omega_j) \)

Objective: Evaluate performance of control laws

Performance: cost of maintaining synchrony in terms of resistive power losses

- Total power losses over network:
  \[
  P_{\text{loss}} \approx \sum_{i \in E} b_{ij}(\theta_i - \theta_j)^2 = \theta^T L_c \theta,
  \]
- Define performance output \( y = L_c \theta \), then total power loss is squared Euclidean norm of output.
- Recall: If input \( w \) is white noise, then the \( H_2 \)-norm of a stable input-output system can be interpreted as
  \[
  ||H||_2^2 = \lim_{t \to \infty} \mathbb{E} \{ y(t)^T y(t) \}
  \]

Expected transient power losses are now given by the systems input-output \( H_2 \)-norms.

Main result: DAPI control improves performance

- Assume uniform resistance-to-reactance ratios \( \alpha = g_i/b_i \)
- Let \( \lambda_n, n = 2, \ldots, N \) be the positive eigenvalues of \( L_d \)

**Theorem**

\[
||H_{\text{droop}}||_2^2 = ||H_{\text{DAPI}}||_2^2 = \frac{\alpha}{2k}(N - 1)
\]

\[
||H_{\text{DAPI}}||_2^2 = \frac{\alpha}{2k} N \sum_{n=2}^{N} \frac{1}{\gamma^2 (\gamma^2 + \frac{1}{\tau_n^\alpha})^2}
\]

It holds \(||H_{\text{DAPI}}||_2^2 < ||H_{\text{DAPI}}||_2^2\), that is, DAPI incurs smaller losses in maintaining synchrony than droop and CAPI control.

Observations:

- CAPI performance independent of network topology and gain \( q \)
- Losses grow with network size \( N \) in both cases
- DAPI performance better for sparse topologies
- Optimal value for distributed averaging parameter \( \gamma = c_i/b_i \) is small.

Interpretation: Emulation of self-damping

- DAPI control
  
  \[ \dot{\omega}_i \approx [\text{droop control}] + \frac{1}{q} \int_0^r \omega(\tau) d\tau - \omega_i \]
  
  - Reduces need for power flows for disturbance attenuation, hence lower associated costs
  - Distributed averaging of state \( \Omega \) required for stability, but too strong gains \( c_i \) reduce self-damping effect

Future work includes: studying optimal topology of secondary controller layer, and alternative performance metrics.